Drag Coefficients for Astronomical Observatory Satellites

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The effects of satellite attitude history on drag coefficients are evaluated by analytic averaging over the orbital motion. This permits translating the results of a free molecular drag analysis into mission-specific orbit decay rates. The type of mission considered is an astronomical observatory satellite in a circular orbit, inertially pointing its telescope axis toward different regions of the celestial sphere. Average drag coefficient is shown to undergo wide variations during such a mission, because of the large satellite aspect ratio. Numerical results are given for the High Energy Astronomy Observatory-B X-ray telescope, and the relation of its drag coefficient to galactic belt mapping is discussed. The long term drag coefficient is shown to decrease at high orbit inclinations for this class of missions.

Nomenclature

A = constant reference area associated with C_D
$b_0, b_1, b_2 \ldots,$ = curve fit coefficients for $C_D(\alpha)$
$C_D(\alpha)$ = observatory drag coefficient, averaged over roll attitude
$<C_D>$ = C_D averaged over one complete orbit
i = observatory orbit inclination relative to Earth's equatorial plane
i_G = orbit inclination relative to the galactic plane
$\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3$ = direction cosines of south galactic pole, in equatorial coordinates
u = modified argument of latitude for observatory position
α = observatory angle of attack relative to velocity vector
$\alpha_\odot$ = solar right ascension
β = angle between sun line and the ζ axis
δ = solar declination
ν = rotation angle of X-axis in the solar normal plane
ν^* = angle between η axis and intersection of galactic and solar normal planes
ξ, η, ζ = Earth-centered coordinate system used for averaging C_D
ψ = angle between the galactic and solar normal planes
Ω_0 = initial right ascension of observatory orbit node
Υ = direction of vernal equinox
λ_G = maximum galactic latitude being observed

Introduction

ORBIT decay rate prediction requires an estimate of satellite drag. The first step in calculating the latter is a free molecular flow analysis, accounting for the satellite's shape and surface properties. Curves are generated giving C_D vs angle of attack, for various roll attitudes. However, for some satellite configurations the drag varies by a factor of 2 or 3 as a function of attitude.

The mission analyst must use the aerodynamicist's curves to predict orbit decay rate as a function of time after launch. The present paper describes an analytical procedure for evaluating mission-specific attitude history effects on the resulting satellite drag.

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Curve Fitting the Aerodynamicist's Results

Astronomical observatory satellites, such as the High Energy Astronomy Observatory (HEAO) and the Large Space Telescope (LST), tend to have a large ratio of length/diameter. This results from long telescope focal lengths combined with launch vehicle fairing diameter constraints. For these satellites the dependence of C_D (for a fixed reference area, A) on roll attitude is small compared to the angle-of-attack dependence. An arithmetic averaging of C_D over roll attitude (i.e., about the telescope axis) can therefore be used to produce a composite drag curve, corresponding to an equivalent shape having cylindrical symmetry.

A representative curve of C_D vs angle of attack is shown by the solid curve in Fig. 1. This corresponds to the HEAO-B X-ray telescope configuration shown in Fig. 2. The free molecular drag analysis on which Fig. 1 is based assumed normal and tangential momentum coefficients of unity. Molecules were assumed to be diffusely re-emitted from the surface, with full accommodation. Note that $C_D(\alpha)$ varies from 3.4 from an "end-on" attitude to 9.0 for broadside orientation. This type of curve is, in general, not symmetrical about $\alpha = 90°$. The problem at hand is to determine an appropriate average, $<C_D>$, for the orbit parameters and sky coverage objectives of the scientific mission.

Orbit lifetime prediction programs often work with an atmospheric density averaged over the diurnal bulge effects. This is combined with an average ballistic coefficient, W/C_DA, to calculate the smoothed altitude decay rate. The $<C_D>$ results derived in this paper are well-suited for use in orbit lifetime programs.

Observatory angle of attack can vary over a single orbit by as much as $0 > \alpha > 180°$ during inertial pointing. The analytical approach used here is to approximate $C_D(\alpha)$ by a simple curve fit, and then average over the orbit to remove the short-term variation. The adopted curve fit is of the form

$$C_D(\alpha) = \sum_{j=0}^{n} b_j \cos^j \alpha \tag{1}$$

Note that

$$b_o = C_D(90°) \tag{2a}$$

$$\sum_{j=0}^{n} b_j = C_D(0) \tag{2b}$$

$$\sum_{j=0}^{n} (-1)^j b_j = C_D(180°) \tag{2c}$$

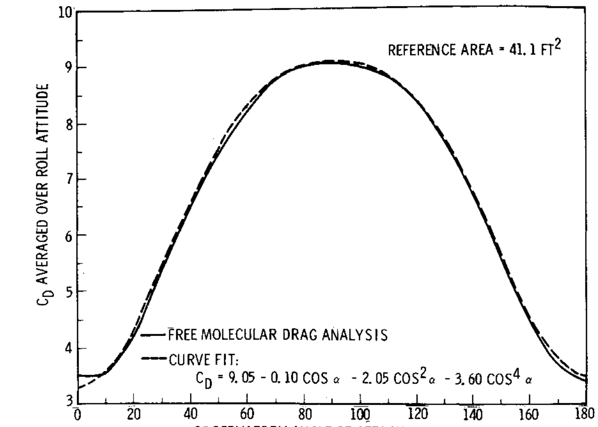

Fig. 1 HEAO-B C_D (α) averaged over roll angle.

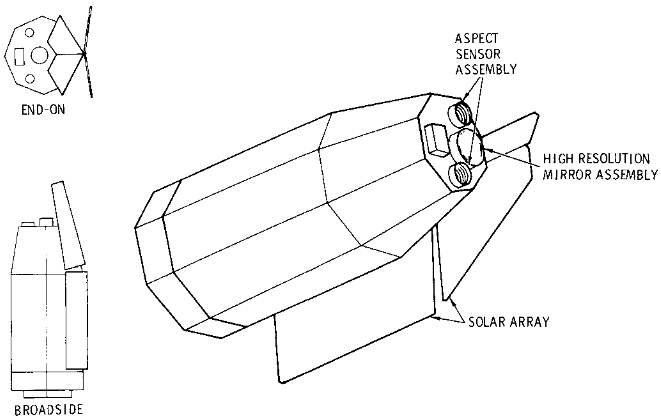

Fig. 2 HEAO-B observatory configuration used for C_D (α).

The nonsymmetry of C_D about $\alpha = 90°$ is only slight for most satellites, so that $|b_1/b_0|, |b_3/b_0|$, etc. are small. It will be shown that the odd powers of cos α in Eq. (1) average out and make no contribution to $<C_D>$. The ratios $|b_2/b_0|$, etc. tend to increase with observatory length/diameter.

The dashed curve in Fig. 1 shows the numerical accuracy provided by the approximation

$$C_D(\alpha) = 9.05 - 0.10\cos\alpha - 2.05\cos^2\alpha - 3.60\cos^4\alpha \tag{3}$$

This degree of curve fit accuracy is adequate, because of basic uncertainties in the free molecular flow analysis.

Dependence of C_D on Orbital Parameters

An observatory telescope axis points inertially for various intervals toward different regions of the celestial sphere. The telescope axis (X-axis) is assumed here to remain in the plane normal to the sun line (referred to as the solar normal plane) because of electrical power and thermal considerations. The inertial orientation of the solar normal plane varies with calendar date, permitting observation of every region in the celestial sphere within a 6-month interval. The pointing history must be selected in accordance with the sky coverage objectives of the particular mission.

C_D, as approximated by Eq. (3), depends only on X-axis orientation relative to the velocity vector; i.e., on α. The coordinate system used here for averaging C_D over the orbital motion is similar to that used in Ref. 1. Figure 3 shows the noninertial ξ,η,ζ coordinate system, chosen such that the observatory orbit always lies in the ξ,η plane. Instantaneous observatory orbital position is defined by a modified argument

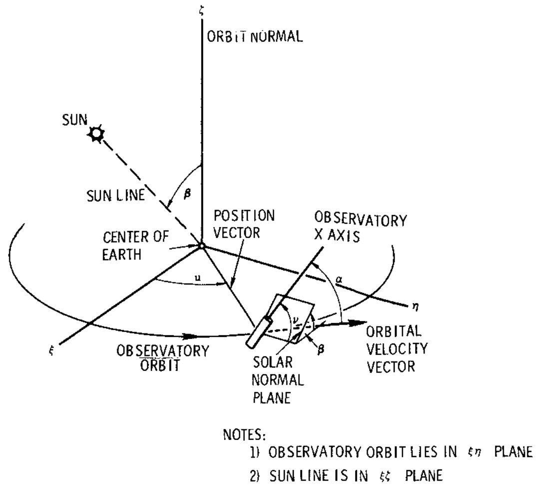

Fig. 3 Coordinates used for orbital average of C_d (α).

of latitude, u. The ξ,ζ plane always contains the sun line. Also shown is the angle-of-attack, α, between the velocity vector and the telescope axis, denoted X.

Let ν denote the counterclockwise angle, measured in the solar normal plane, from the ξ,η plane to the observatory X-axis. The desired sky coverage can be obtained by choosing the proper value (s) of ν on each day of the mission. Sustained inertial pointing toward a given localized region of the sky requires ν to be held relatively fixed for several orbits at a time.

Let β denote the angle between the sun line and ζ axis. Neglecting the small effect of atmospheric rotation on the observatory angle-of-attack,

$$\cos\alpha = \cos\beta\sin\nu\,\sin u + \cos\nu\,\cos u \tag{4}$$

The expression for sin α, analogous to Eq. (4), involves the square root of a function of β,ν, and u. Orbital averages of such terms would produce a more complicated analytical result, without significant improvement in numerical accuracy of $<C_D>$. For this reason, the adopted curve fit contains only powers of cos α.

Average of C_D Over an Orbit Revolution

The dependence of C_D on β, ν, and u is given by Eqs. (1) and (4). During each orbit ($0 \le u < 2\pi$) cos α will oscillate between the following extrema:

$$(\cos\alpha)_{\max,\min} = \pm(\cos^2\nu + \cos^2\beta\sin^2\nu)^{1/2} \tag{5}$$

The change in β during one orbit is negligible. Furthermore, ν is assumed to remain relatively constant for several orbits, to permit sustained observation of a localized region of the sky. Hence the average C_D over the orbital motion ($0 \le u < 2\pi$), holding β and ν constant is

$$+ \tfrac{3}{8} b_4 (\cos^2\beta\sin^2\nu + \cos^2\nu)^2$$
$$+ 5/16\, b_6 (\cos^2\beta\sin^2\nu + \cos^2\nu)^3 \tag{6}$$

The curve fit terms b_1 cos α, $b_3\cos^3\alpha$, $b_5\cos^5\alpha$, etc. make no contribution to $<C_D>$, because they depend only on odd functions of sin u and cos u.

For the curve fit in Fig. 1, it was found that $b_2, b_4 < 0$, and $b_6 = 0$. For this case simple expressions can be derived from Eq. (6) for upper and lower bounds on $<C_D>$, assuming that X-axis remains in the solar normal plane. The maximum possible $<C_D>$ is

$$<C_D>_{\max} = b_o + \tfrac{1}{2} b_2 \cos^2\beta + \tfrac{3}{8} b_4 \cos^4\beta \tag{7}$$

and occurs for $\cos\nu = 0$. This corresponds to pointing the telescope axis at the maximum possible angle, β, relative to the orbit plane. Note that $<C_D>_{max}$ is less than the broadside value b_o, except when $\cos\beta = 0$.

The minimum possible $<C_D>$ from Eq. (6) is

$$<C_D>_{\min} = b_o + \tfrac{1}{2} b_2 + \tfrac{3}{8} b_4 \tag{8}$$

and occurs for $\sin\nu = 0$. This corresponds to pointing the X-axis along the intersection between the orbit plane and the solar normal plane (i.e., along the η axis). $<C_D>$ could also achieve the same minimum value if the telescope could be pointed along any direction in the orbit plane.

Numerical results calculated from Eqs. (7) and (8) show that a time average on these maximum and minimum values over a 12-month period could be anywhere between 6.68 and 8.85, for an equatorial inclination of 22.8°. If all other orbital conditions are the same, the corresponding maximum decay rate would be 32% greater than the minimum.

This C_D averaging method has also been applied to a spacecraft which varies appreciably in inertial attitude during an orbit. HEAO-A experiment objectives require a celestial scan motion (about the sun line) with a period approximately one-third the orbit period. A double-averaging process was used, first over the spacecraft spin cycle and then over the orbital motion. Discussion of those results is beyond the scope of this paper.

$<C_D>$ Histories for Hypothetical Pointing Missions

It is illustrative to define a few representative pointing histories which appear to be of scientific interest, and calculate numerical results for the corresponding $<C_D>$ histories.

Galactic Belt Coverage Geometry

Figure 4 shows a unit celestial sphere with the north ecliptic pole at the top. The outside surface of the sphere is shown, with earth at its center. The X-axis is assumed to remain in the solar normal plane. Also shown are the ξ, η, and ζ axes from Fig. 3. The observatory orbit normal, ζ, precesses about the Earth's north pole, in a cone of half-angle i.

Galactic plane orientation is of particular interest to HEAO-B because many known X-ray sources are located at relatively low galactic latitudes. As shown in Fig. 4, the galactic plane is inclined to the ecliptic by approximately 60.2°; the line of intersection is approximately 89° east of the vernal equinox, measured in the ecliptic plane. The shaded region of Fig. 4 represents the galactic "belt."

The orbit plane inclination, i_G, relative to the galactic plane is given by

$$\cos i_G = -l_1 \sin i \sin\Omega + l_2 \sin i \cos\Omega - l_3 \cos i \tag{9}$$

geometry is shown in Fig. 5. The angle, ψ, between the galactic and solar normal planes is given by

$$\cos\psi = l_1 \cos\delta \cos\alpha_\odot + l_2 \cos\delta \sin\alpha_\odot + l_3 \sin\delta \tag{10}$$

$$\sin\psi = +(1 - \cos^2\psi)^{1/2} \tag{11}$$

Solar right ascension and declination are represented by $\alpha_\odot$ and δ, respectively.

The great circle arc, ν^*, between points "η" and "2" is given by

$$\cos\nu^* = (\cos\beta \cos\psi + \cos i_G)/\sin\beta \sin\psi \tag{12}$$

For each solar right ascension, there are two angular locations 180° apart (labelled "2" on Fig. 4) where the telescope axis can point at zero galactic latitude while remaining normal to the sun line. Nonzero galactic latitudes may be observed by shifting the observatory axis away from location 2, along the solar normal plane.

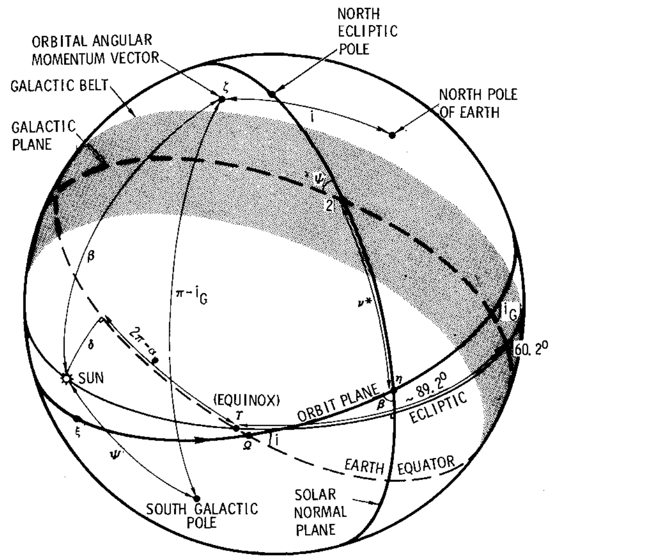

Fig. 4 Geometry which relates celestial pointing and drag history.

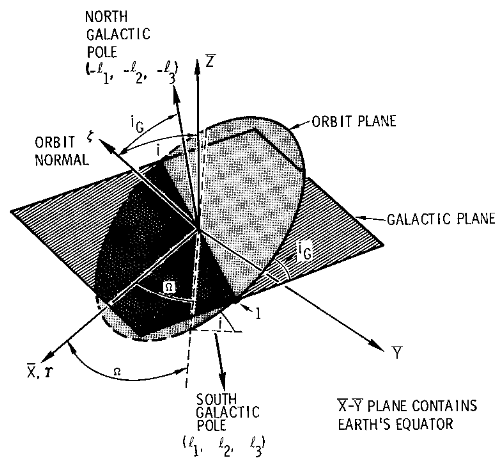

Fig. 5 Orbit inclination, i_G, relative to galactic plane.

$<C_D>$ for Representative Pointing Histories

Figure 6 shows the time variations of $<C_D>$ for hypothetical missions in which the observatory X-axis points along the intersection between the galactic and solar normal planes. This corresponds to $\nu = \nu^*$ in Eq. (4). The time between successive extrema of $<C_D>$ is governed mainly by the modal regression rate. Curves are given for orbit inclinations of 0° and 75°. The effect of this large amplitude but slow $<C_D>$ variation on orbit period must be calculated in order to make accurate predictions of observatory rise-set times over ground stations, and to generate reference orbital histories for planning purposes.

Figure 7 shows the dependence of $<C_D>$, averaged over 12 months, on orbit inclination. The value in Fig. 7, for a given inclination, is the arithmetic average of $<C_D>$ computed at 10-day intervals. For low and moderate inclinations, this could also be obtained by analytic averaging of Eq. (6) over the orbit nodal regression relative to the sun line. For a 22.75° inclination, $<C_D>$ for a worst-case pointing history would be 8.85, or 32% greater than for a minimum-drag pointing history. These results are a measure of the long-

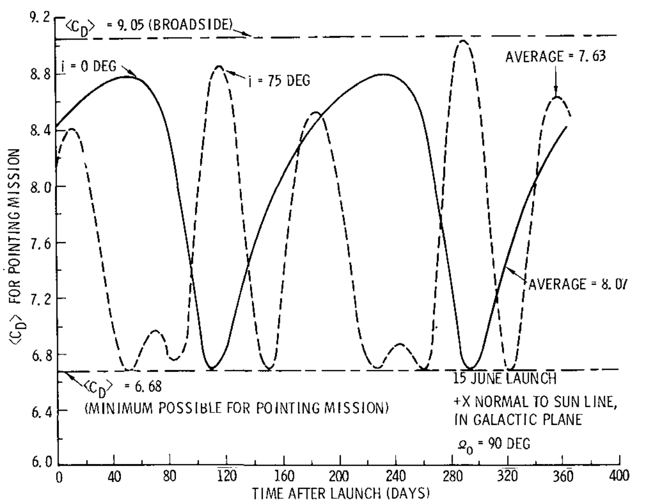

Fig. 6 Galactic pointing drag histories for two orbit inclinations.

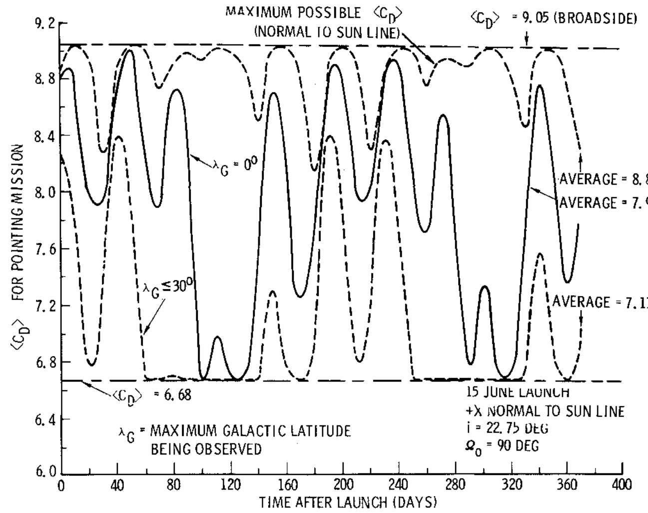

Fig. 8 Reduction of $<C_D>$ by off-pointing from galactic plane.

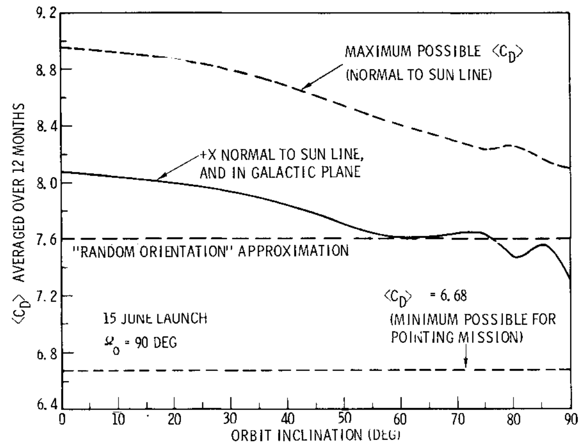

Fig. 7 Inclination dependence of long-term $<C_D>$ average.

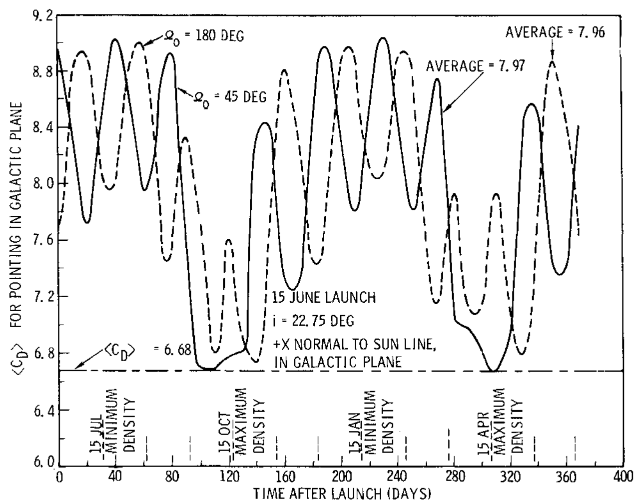

Fig. 9 Use of Ω_0 to time-phase the $<C_D>$ extrema.

clination, similar to the result found in Ref. 1 for planar oriented solar arrays. Different values of Ω_0 produce some variation in the $<C_D>$ average at high inclination, because the reduced nodal regression rate provides less averaging over orbit orientation.

Also shown in Fig. 7 is the "random orientation" result, $<C_D> = 7.60$. This is based on a different analytic average over angle of attack, and assumes that all points on the unit sphere centered at the observatory position are equally probable locations for the relative velocity vector. The random orientation result tends to underestimate drag for low orbit inclinations, as noted in [1] for oriented solar arrays.

If it were essential to reduce $<C_D>$, this could be accomplished by a departure from X-axis pointing at the galactic plane, assumed in Figs. 6 and 7. The lower dashed curve in Fig. 8 represents a hypothetical mission at 22.75° inclination in which $<C_D>$ is reduced as much as possible while always pointing at galactic latitudes less than ±30°. The 12-month average of this curve is 7.11. Also shown in Fig. 8 is the $i = 22.75°$ curve analogous to Fig. 6, which has a 12-month average of 7.98. Hence an 11% reduction in average $<C_D>$ could be obtained while always viewing less than 30° from the galactic plane.

$<C_D>$ can be minimized, while maintaining the telescope axis normal to the sun line, by pointing along the η axis. However, this would produce excessive sky coverage at high latitudes and insufficient coverage at low latitudes. The η axis would be a good "attitude hold" orientation, in the sense of minimizing orbit decay.

Time Phasing of $<C_D>$ and Density Variations

It was previously mentioned that the effect of Ω_0 (launch time) on the 12 month average of $<C_D>$ is relatively small, except for $i \to 90°$. However, Figs. 6,8, and 9 indicate that $<C_D>$ does vary considerably during 12 months even for low inclinations, and that the $<C_D>$ a specific time after launch is quite dependent on Ω_0 and inclination.

Atmospheric density at a given altitude undergoes a semi-annual variation, with maxima occurring in April and October, and minima in January and July.[2] The relative time phasing of the density and $<C_D>$ extrema can have a major effect on week-to-week orbit decay rate. A June 15 launch was assumed in Fig. 9. The $<C_D>$ maxima occur when density is minimum, for both values of Ω_0 shown here. However, $\Omega_0 = 45°$ produces a much broader $<C_D>$ minimum during the seasons of maximum density, thereby reducing the average decay rate during these two seasons. The previous heuristic discussion suggests that long-term altitude decay rates are more sensitive to Ω_0 and inclination than is suggested by Fig. 9. This would make an interesting extension of the present results.

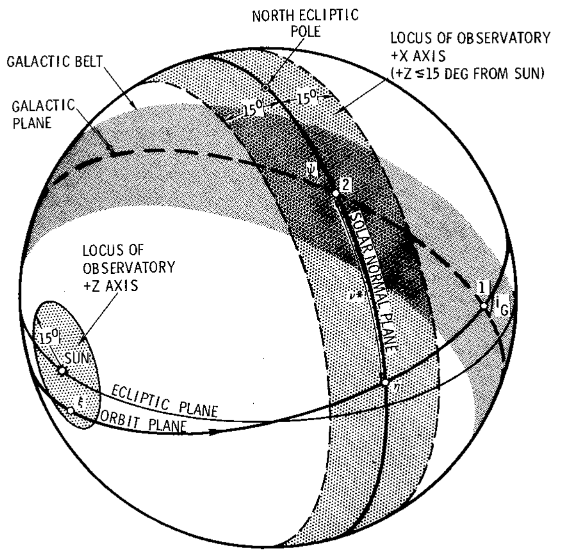

Fig. 10 Sky coverage and $<C_D>$ reduction by departing from solar normal plane.

Coverage Flexibility for X-axis Off-Pointing

Figure 10, similar to Fig. 4, indicates the increased sky coverage available if the X-axis departs from the solar normal plane. HEAO-B will have the capability to point up to 15° off the sun line indefinitely. The accessible sky coverage swath on a given day is indicated by the shaded region adjacent to the solar normal plane in Fig. 10; the accessible portion of the galactic belt, on that day, is the approximate parallelogram-shaped region. By shifting the observatory X-axis off the solar normal plane and toward point "1," $<C_D>$ could be reduced from the numerical values presented in Figs. 6-8, while simultaneously permitting coverage of the galactic plane. This represents an additional degree of freedom in optimizing the overall mission.

Conclusions

It has been shown that average drag coefficient, and hence orbit decay rate, undergoes significant variations during an inertially pointed observatory mission. In addition to the orbit orientation relative to the sun line, this variation depends on the spacecraft attitude history, and hence on the particular observational mission profile. For a 22.75° inclination, average drag for a worst-case pointing history would be 32% greater than for a minimum-drag pointing history. A knowledge of drag coefficient vs time after launch, as provided by this analysis, is necessary for accurate predictions of rise/set times over ground stations and for generating reference orbital histories. In addition to the High Energy Astronomy Observatory -B, these analytical results will be useful for the Large Space Telescope program.

For a given observatory configuration, the long term average drag coefficient decreases at higher orbit inclinations. The dependence on ascending node is small at low inclinations, but increases at higher inclinations due to the reduced nodal regression rate. The observed orbit decay rate will depend in part on the time phasing of drag coefficient extrema relative to seasonal fluctuations in atmospheric density. A similar analytic averaging technique can be used to calculate drag coefficient for a spacecraft which continously spins about the sun line (e.g., HEAO-A) rather than pointing inertially.

References

[1] R. R. Williams, "Atmospheric Drag Cross Section of Spacecraft Solar Arrays," *Journal of Spacecraft and Rockets,* Vol. 9, Aug. 1972, pp. 565-566.

[2] L. G. Jacchia, "New Static Models of the Thermosphere and Exosphere with Empirical Temperature Profiles," Smithsonian Astrophysical Observatory Special Rept. 313, May 6, 1970.